Guidance Methods for Accurate In–Flight Alignment of Navy Theatre Wide Missiles

15 May 2001

Ernest J. Ohlmeyer
Craig Phillips
David Hanger
Mark Jones
Thomas R. Pepitone

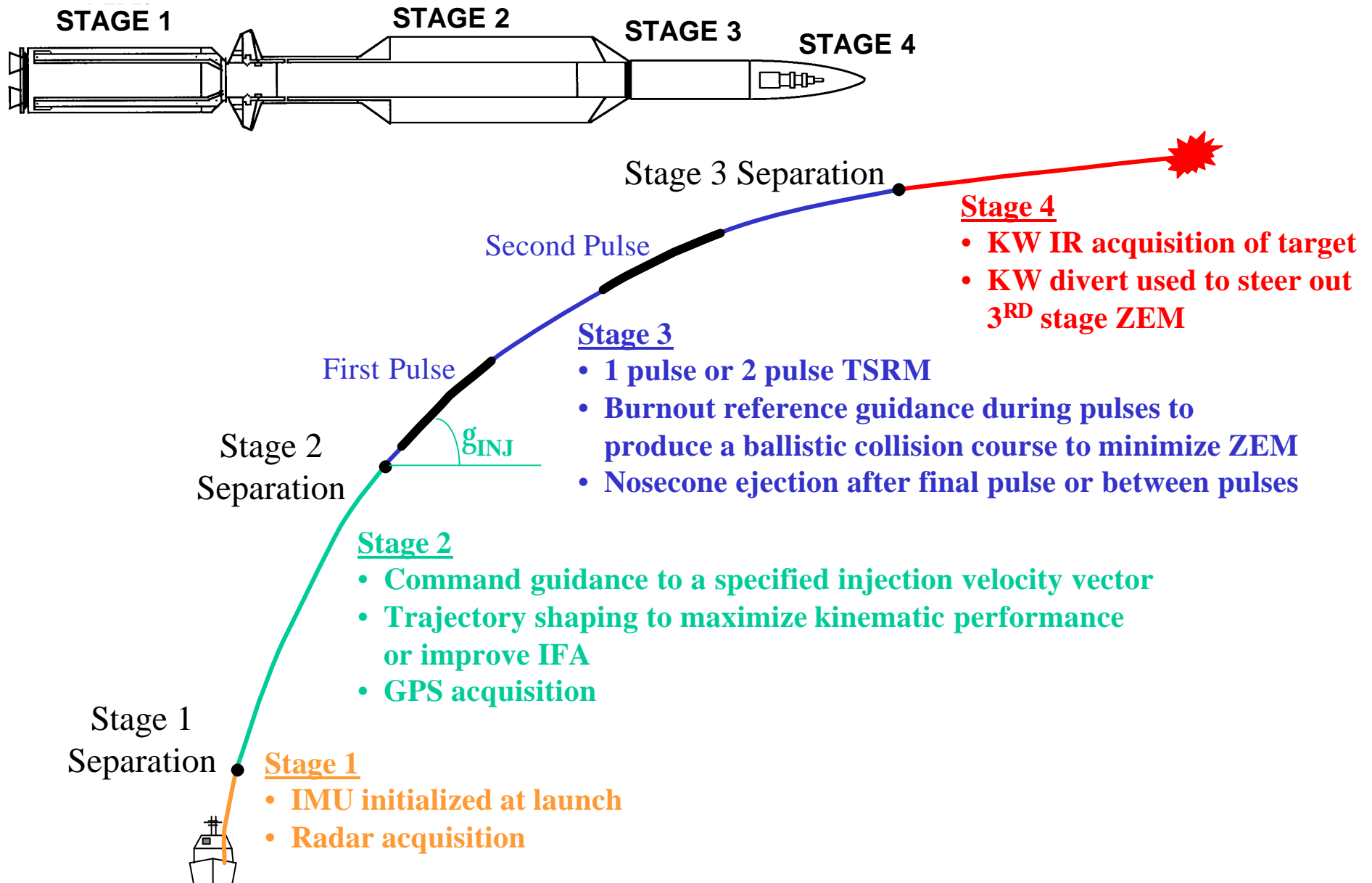**NDIA Missiles &Rockets Symposium and Exhibition**

OUTLINE

- Background

- ADOP - In-Flight Alignment Metric

- Second Stage Guidance Methods

- In-Flight Alignment Analysis

- Summary

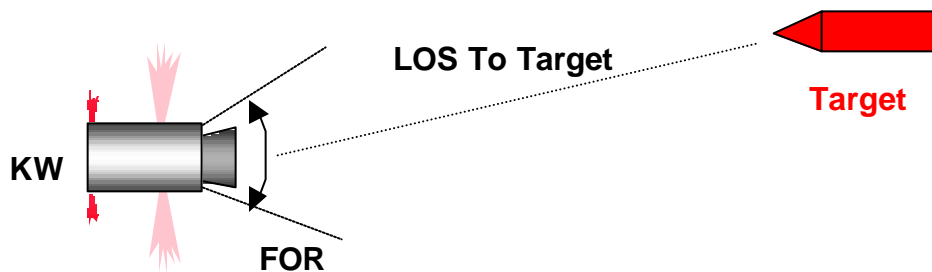# Background

NTW Concept of Operation

STAGE 1 STAGE 2 STAGE 3 STAGE 4

Stage 3 Separation

Stage 4
- **KW IR acquisition of target**
- **KW divert used to steer out 3RD stage ZEM**

Second Pulse

First Pulse

g_{NJ}

Stage 3
- **1 pulse or 2 pulse TSRM**
- **Burnout reference guidance during pulses to produce a ballistic collision course to minimize ZEM**
- **Nosecone ejection after final pulse or between pulses**

Stage 2 Separation

Stage 2
- **Command guidance to a specified injection velocity vector**
- **Trajectory shaping to maximize kinematic performance or improve IFA**
- **GPS acquisition**

Stage 1 Separation

Stage 1
- **IMU initialized at launch**
- **Radar acquisition**

Successful Intercept Requirements

Pointing Requirement

LOS To Target

Target

KW

FOR

- At kinetic warhead (KW) separation the target must be within the seeker field of regard (FOR)

Divert Requirement

INTERCEPT X

Target

KW

KW DIVERT CAPABILITY
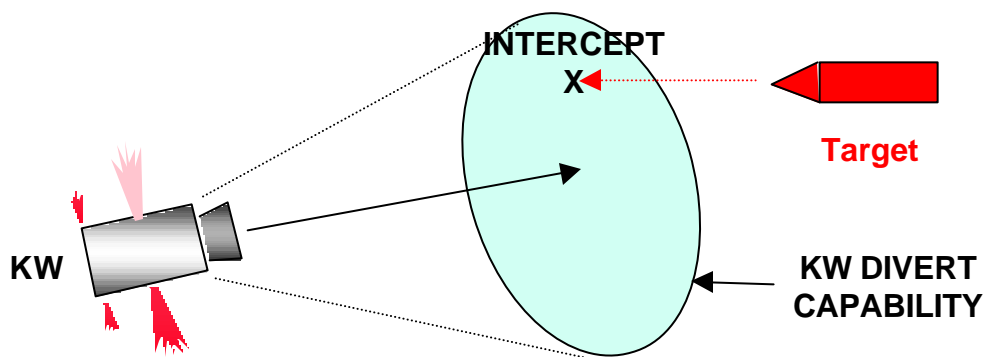
- The zero effort miss (ZEM) must be within the kinetic warhead divert capability

- The missile IMU alignment with respect to the ship defined navigation (ECEF) coordinate frame may have a large unknown error at launch (up to 26 mrad)

  - This error dominates the error budget and degrades performance

  - The in-flight alignment (IFA) process calibrates the IMU alignment with respect to the navigation coordinate frame during flight

  - An integrated GPS/IMU missile navigation system was first used on Standard Missile to perform this in-flight alignment as part of the Terrier LEAP experiment

In order To Meet The Pointing Error Allocation The Missile Initial Attitude Error Must Be Reduced Inflight

How Does Inflight Alignment Work?

- Background Facts

  - The major alignment error component to be calibrated is the IMU alignment with respect to the navigation frame (≤ 26 mrad)

  - When accelerations are transformed with an IMU alignment error to the navigation frame an acceleration error develops

- The Aiding Process

  - Acceleration errors, when integrated, result in velocity errors which result, in turn, in position errors

  - Navigation errors are observable by comparing inertial navigation estimates of the position and velocity to measurements from outside sources:

    - Radar measurements (position)

    - GPS measurements (position & velocity)

  - Errors are corrected via an on-board Kalman Filter

In-Flight Alignment Metric
" ADOP "

- Attitude Dilution Of Precision (ADOP) was developed as a trajectory induced observability metric of in-flight alignment

- There are two fundamental ingredients in the ADOP metric

    - The missile acceleration time profile

    - The GPS and radar measurement noise error time profiles

- An interpretation of the ADOP metric

    - Missile total (RSS) attitude alignment error with respect to the navigation coordinate frame (3-σ value expressed in milli-radians)

    - A value less than 5 mrad is considered good performance and a value greater than 5 mrad is considered degraded performance

ADOP Attributes

- Trajectory induced observability metric for in-flight alignment

- A simplified error model that is economical to run

- Provides lower bound on attitude errors for benchmarking in-flight alignment performance

- Can be used to generate observability maps over the tactical battlespace

- Shows difficult regions of the battlespace for in-flight alignment

ADOP Observability Contour Map Spanning the Battlespace

Better IFA

Poor IFA

Altitude

Ground Range
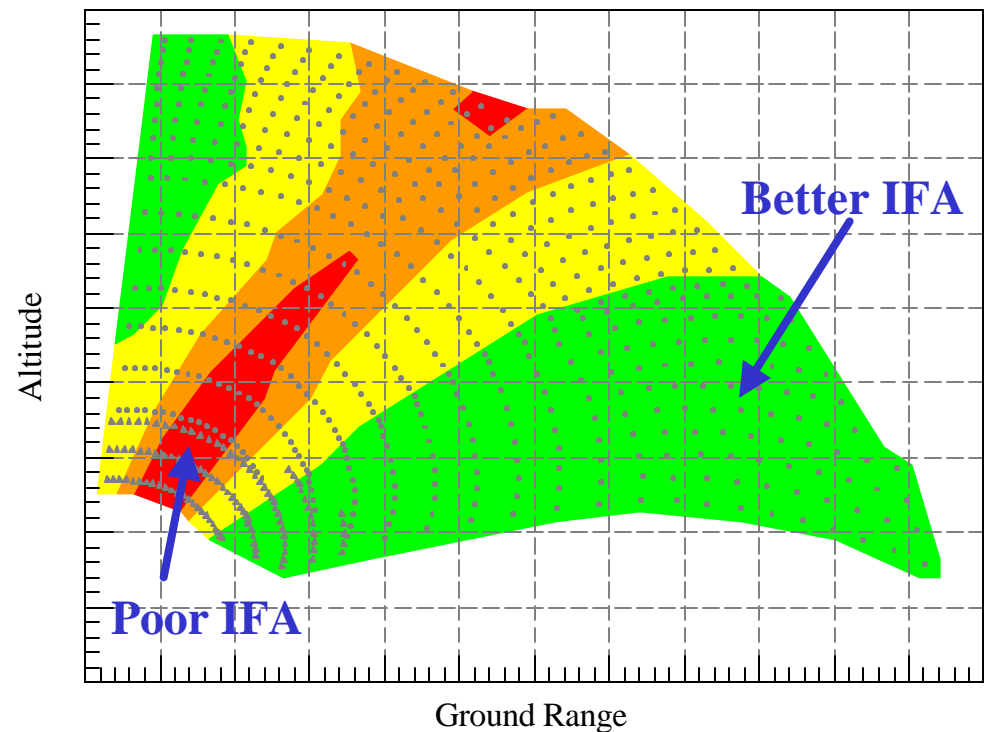
ADOP Values (mrad)

> 20	20 - 15	15 - 10	10 - 5	5 - 0

Second Stage Guidance Methods

- Cross Product Guidance (CPG)
  - Guides to a specified injection velocity vector
  - Approximates an optimal kinematic trajectory

- Delayed Cross Product Guidance (DCPG)
  - Similar to CPG, guides to a specified injection velocity vector
  - Guidance initiation is delayed to improve IFA

- Modified Cross Product Guidance (MCPG)
  - Similar to CPG, guides to a specified injection velocity vector
  - Guidance initiation is delayed
  - Adds a shaping term to improve IFA
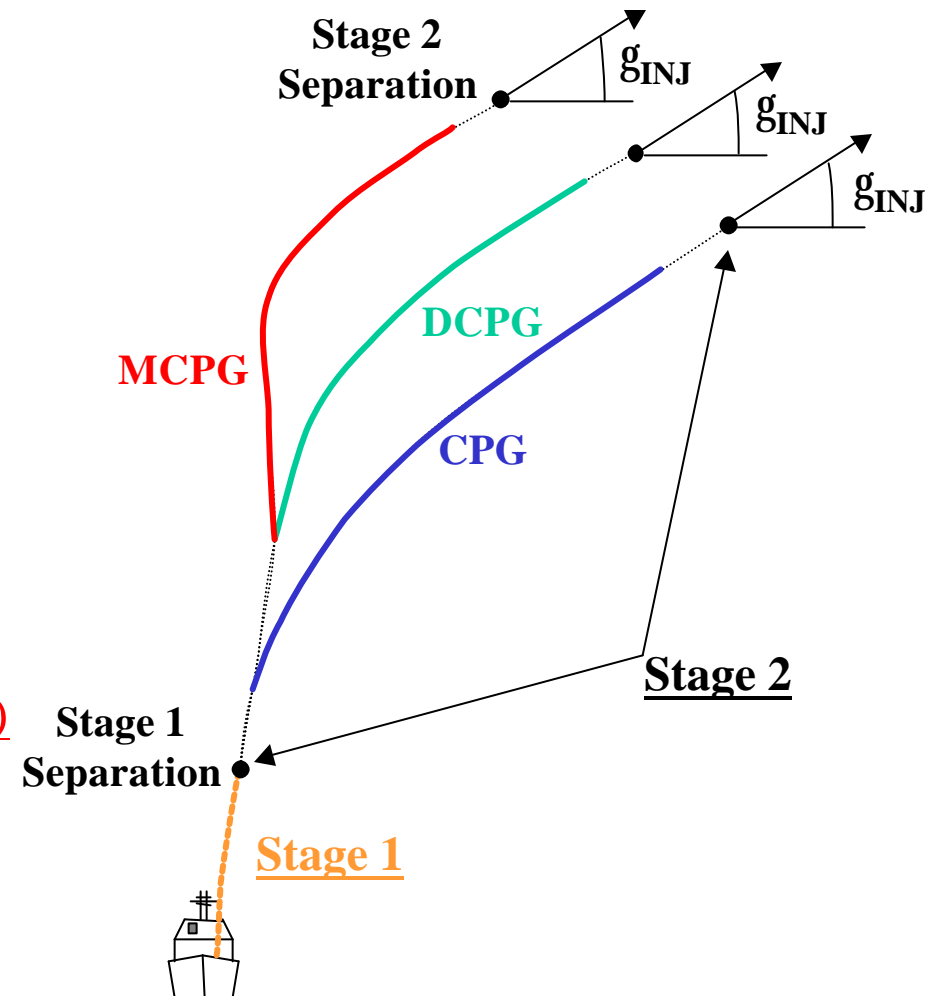
Guidance Law Definitions

$$\vec{A}_C = -K_1 V \sin d \hat{u} + K_2(t)\hat{u}$$

(Cross Product Term) (Shaping Term)

- **Cross Product Term**:
  - CPG, DCPG, & MCPG
  - Nulls heading error and forces convergence to injection velocity vector
  - K_1 gain is scheduled with γ_{INJ} to minimize angle-of-attack

- **Shaping Term**:
  - MCPG only
  - Applies short-lived acceleration in direction opposite to cross product term to induce observability
  - K_2 gain is scheduled with γ_{INJ} to maximize effect in regions of poor IFA
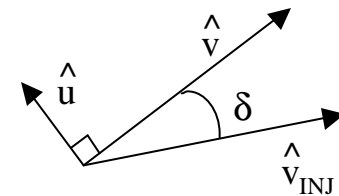
$$\overline{u} = \hat{v} \times (\hat{v} \times \hat{v}_{INJ})$$

$$|\overline{u}| = \sin d$$

$$\hat{u} = \frac{\overline{u}}{|\overline{u}|}$$

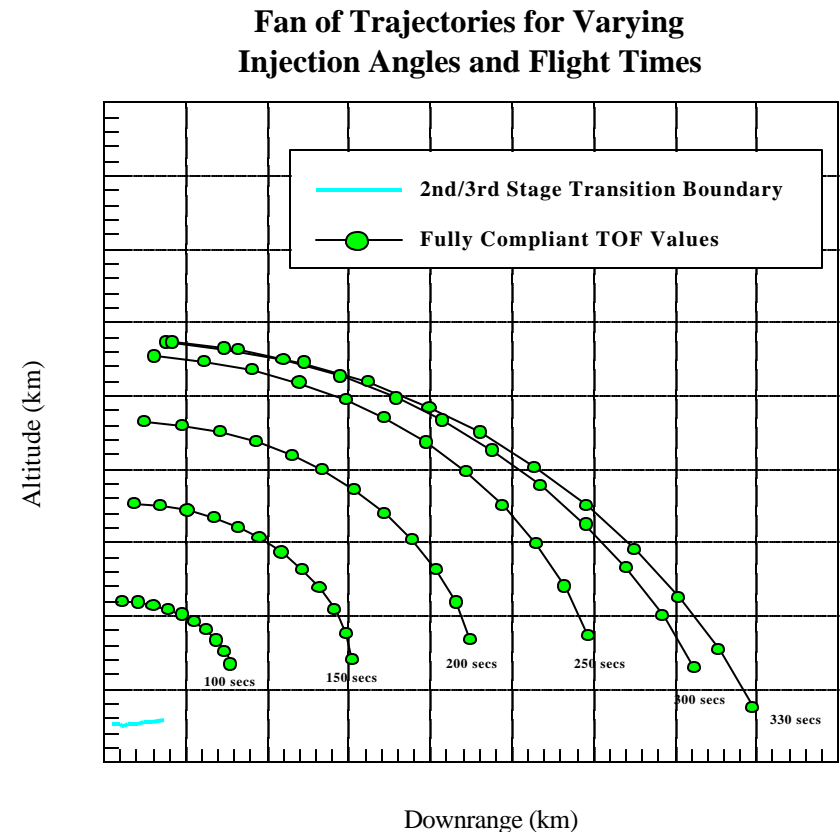|			
$\overline{A}_C$	= commanded acceleration vector	$\hat{u}$	= cross product unit vector
V	= velocity magnitude	d	= angle between $\hat{v}$ and $\hat{v}_{INJ}$
$\hat{v}$	= current velocity unit vector	K_1	= cross product term gain
$\hat{v}_{INJ}$	= commanded injection velocity unit vector	K_2	= shaping term gain

2ᴺᴰ Stage Guidance Attributes

- Second stage guidance is closed-loop

- At lower injection angles, accelerations are limited early in second stage to meet the aero-thermal constraint

- Used to generate a fan of trajectories for varying injection angles and flight times to span the kinematic battlespace

- ADOP measured at various flight times along each trajectory to create observability maps

Fan of Trajectories for Varying Injection Angles and Flight Times

Legend:
— 2nd/3rd Stage Transition Boundary
●— Fully Compliant TOF Values

Axis labels: Altitude (km), Downrange (km)

Trajectory time labels: 100 secs, 150 secs, 200 secs, 250 secs, 300 secs, 330 secs

# IFA Analysis

IFA Analysis for 3 Guidance Laws

- IFA performance measured by ADOP observability maps covering the battlespace

- ADOP maps generated for each guidance law:
  - CPG
  - DCPG
  - MCPG

- ADOP maps examined for two types of aiding:
  - Radar only
  - Radar & GPS

- ADOP maps examined at two trajectory events:
  - $2^{ND}/3^{RD}$ stage separation
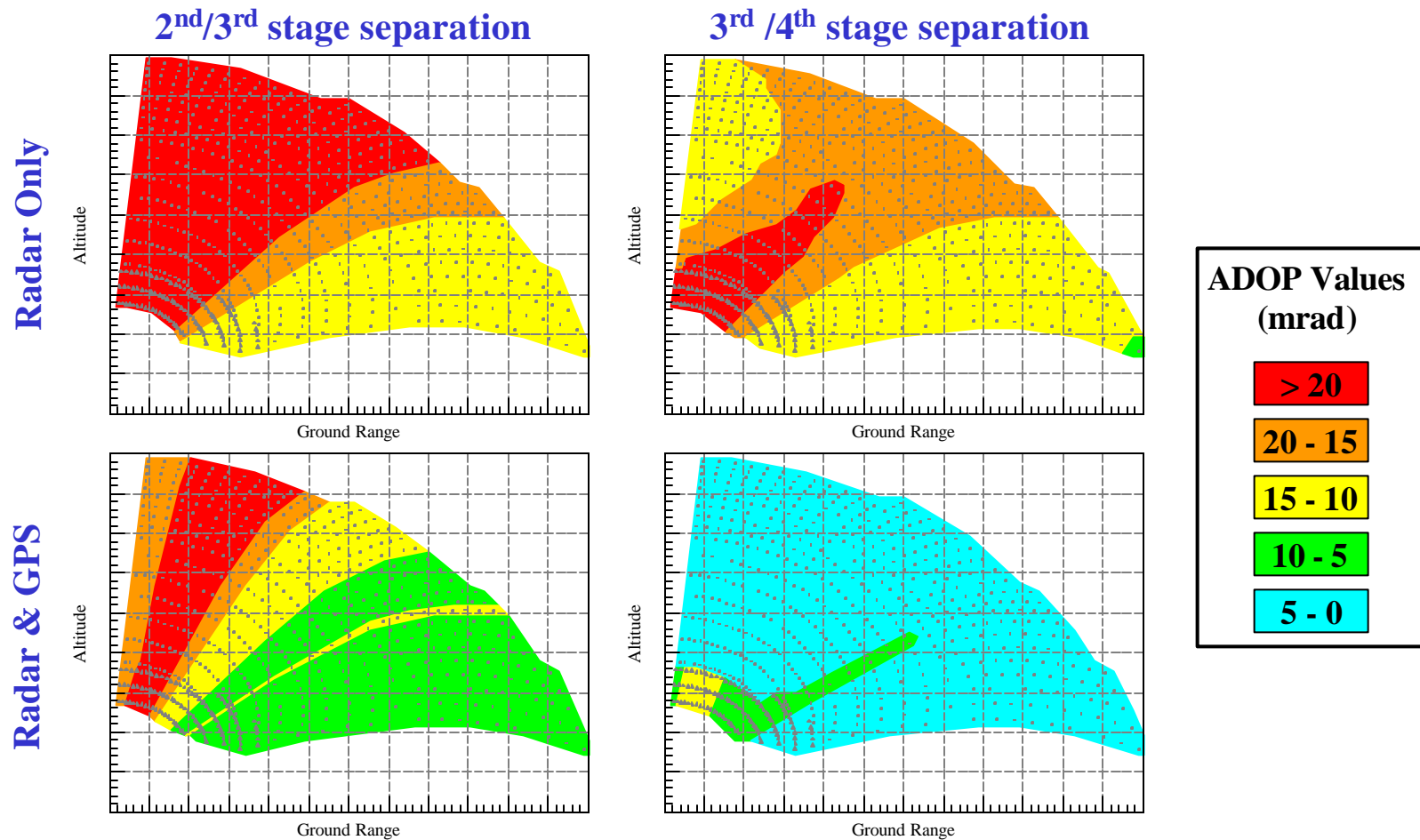 - $3^{RD}/4^{TH}$ stage separation

Example ADOP Histories

ADOP Histories for Radar Only Aiding
for the 3 Guidance Methods
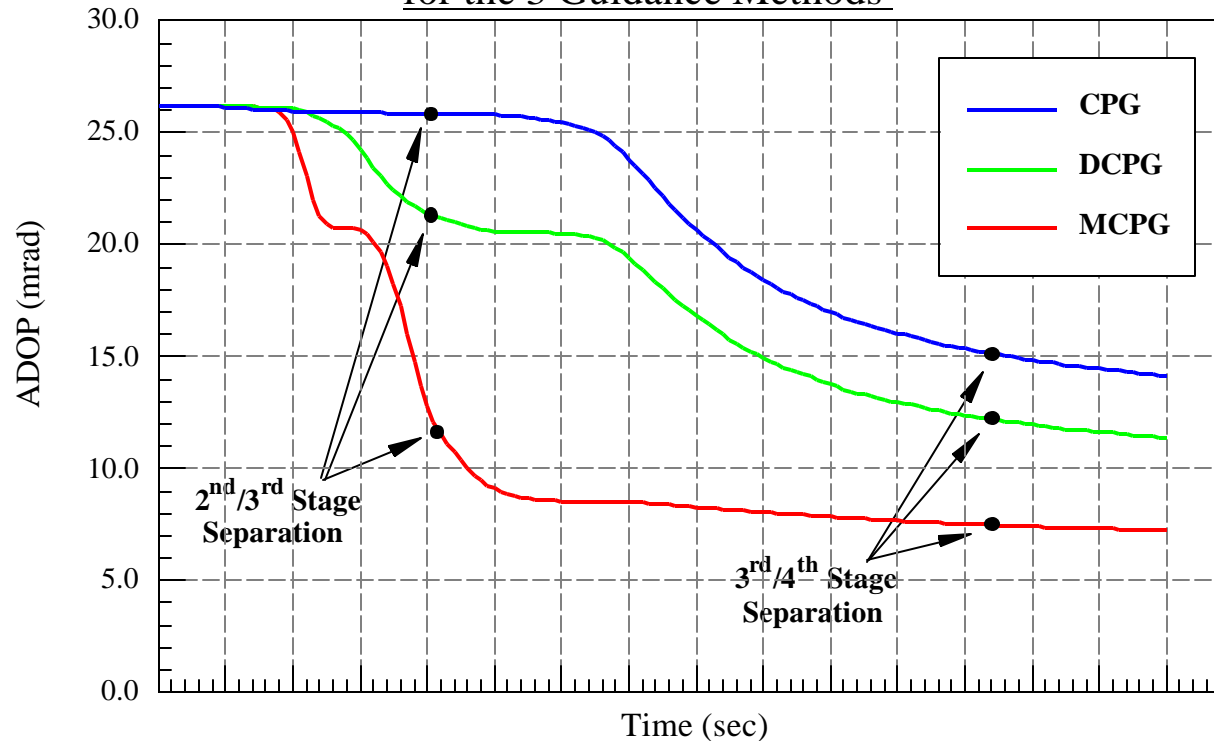
- ADOP time histories show improvement in IFA performance at 3rd/4th stage separation over 2nd/3rd stage separation
- IFA performance improvement at the later flight time results from
  - Additional time for aiding from outside sensors
  - Additional accelerations from the 3rd stage

CPG ADOP Maps

2nd/3rd stage separation

3rd /4th stage separation

Radar Only

Radar & GPS

Altitude

Ground Range

ADOP Values (mrad)

| > 20 |
| 20 - 15 |
| 15 - 10 |
| 10 - 5 |
| 5 - 0 |

- IFA improves from 2nd/3rd stage separation to 3rd/4th stage separation for both aiding methods
- IFA improves for radar & GPS aiding over radar only aiding
- IFA requirement satisfied over majority of the battlespace for the radar & GPS aiding case at the 3rd/4th stage separation point

DCPG ADOP Maps

2nd/3rd stage separation

3rd /4th stage separation

Radar Only

Altitude

Ground Range

Altitude

Ground Range

Radar & GPS

Altitude

Ground Range

Altitude

Ground Range

ADOP Values (mrad)

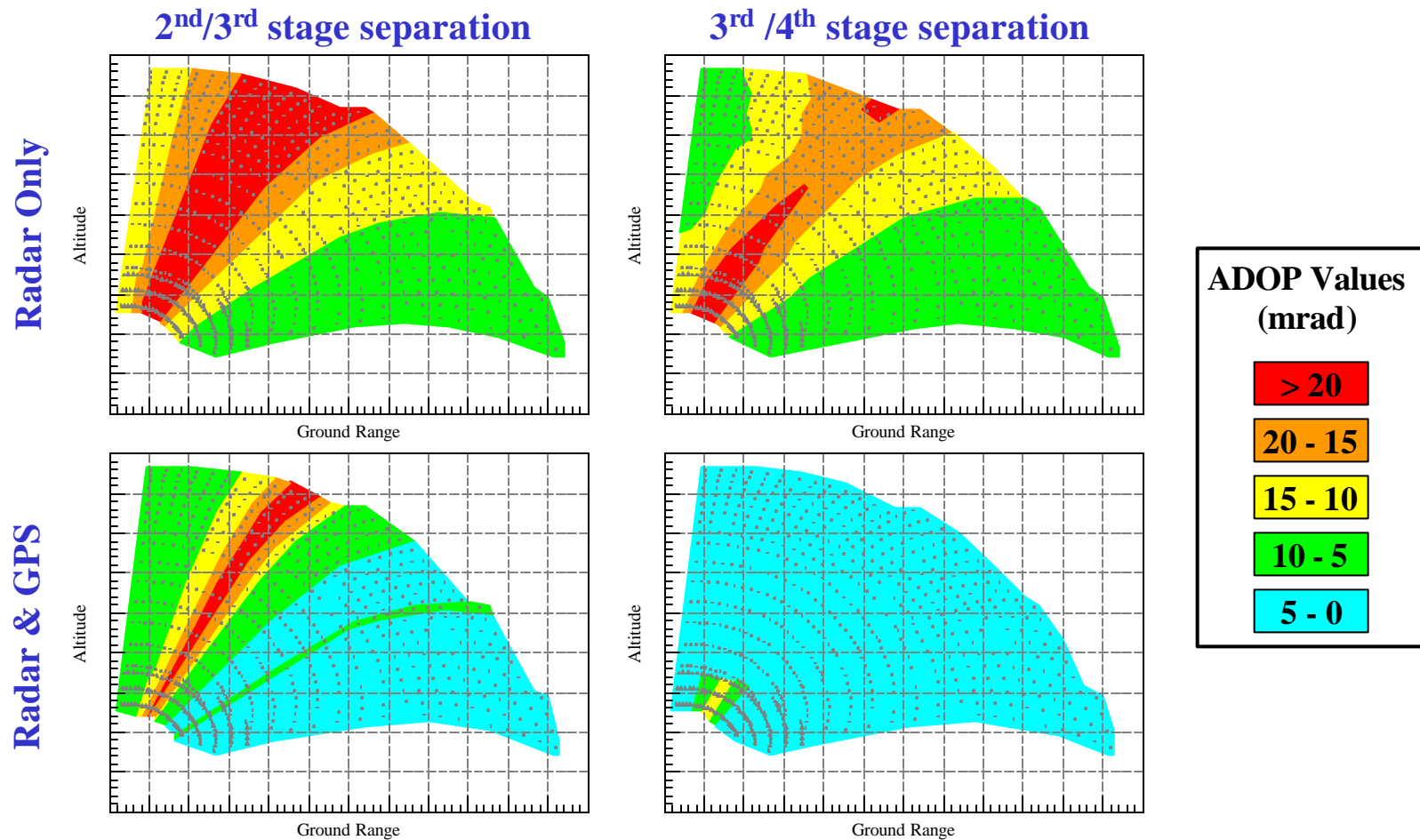| > 20 |
| 20 - 15 |
| 15 - 10 |
| 10 - 5 |
| 5 - 0 |

- IFA improves from 2nd/3rd stage separation to 3rd/4th stage separation for both aiding methods
- IFA improves for radar & GPS aiding over radar only aiding
- IFA requirement satisfied over majority of the battlespace for the radar & GPS aiding case at the 3rd/4th stage separation point
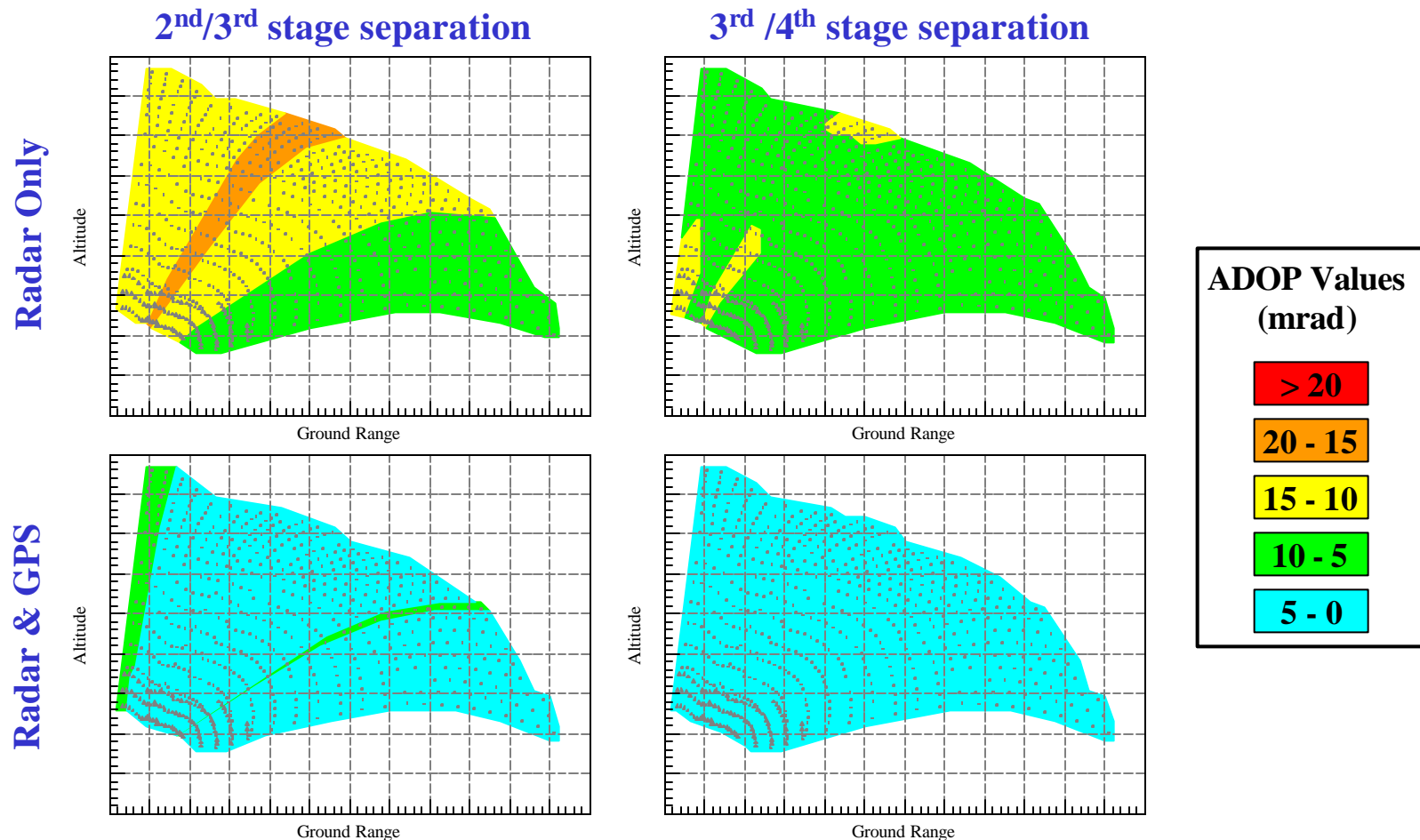
MCPG ADOP Maps

2nd/3rd stage separation   **3rd /4th stage separation**

Radar Only

Altitude / Ground Range

Altitude / Ground Range

Radar & GPS

Altitude / Ground Range

Altitude / Ground Range

ADOP Values (mrad)

| > 20 |
| 20 - 15 |
| 15 - 10 |
| 10 - 5 |
| 5 - 0 |

- IFA improves from 2nd/3rd stage separation to 3rd/4th stage separation for both aiding methods
- IFA improves for radar & GPS aiding over radar only aiding
- For radar & GPS aiding, IFA requirement satisfied over most of the battlespace at 2nd/3rd stage separation and satisfied over the entire battlespace at 3rd/4th stage separation

DCPG and MCPG Kinematic Penalties

Maps of Burnout Velocity
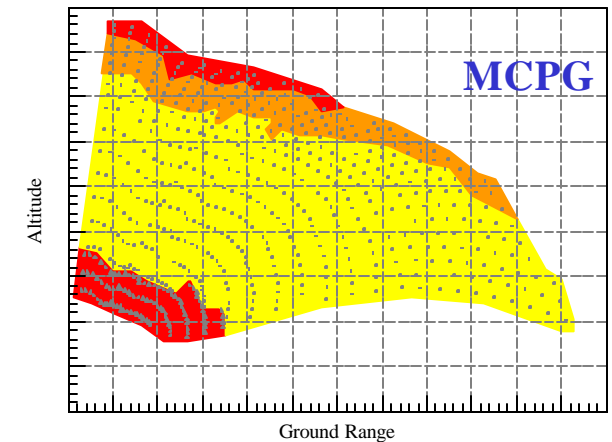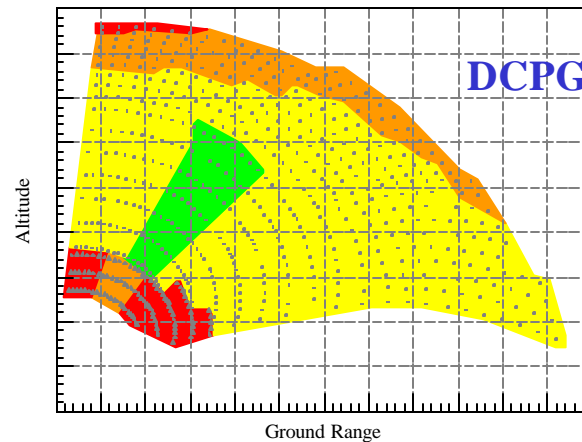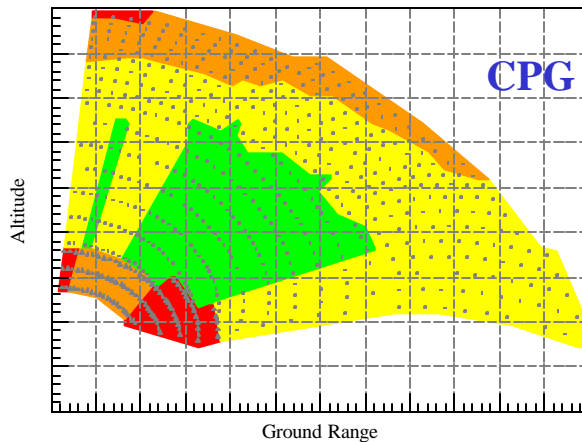
Normalized Velocities

| 0.5 - 0.6 | 0.6 - 0.7 | 0.7 - 0.8 | 0.8 - 0.9 | 0.9 - 1.0 |
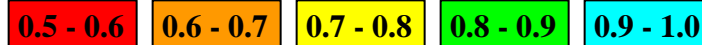
- Battlespace is slightly reduced in ground range with DCPG and further reduced in altitude with MCPG

- Burnout velocities are slightly decreased for DCPG and further reduced for MCPG in the regions of largest trajectory shaping

Summary

- IFA is necessary to meet the KW seeker pointing requirement

- ADOP is the trajectory induced IFA observability metric

- IFA performance has been analyzed for three different second stage guidance laws

- The addition of GPS aiding significantly improves IFA

- The longer aiding period for 3rd/4th stage separation improves IFA over 2nd/3rd stage separation

- Both DCPG and MCPG provide improved IFA performance over CPG

- Using MCPG and with radar & GPS aiding, the IFA requirement is satisfied over the majority of the battlespace at 2nd/3rd stage separation and over the entire battlespace at 3rd/4th stage separation

- Both burnout velocity and the overall battlespace are slightly reduced for DCPG and MCPG

Backup Slides

Example Pointing Error Allocation

Assumptions:
- Pr target within radius = **0.9974**
- V_C = **4068** m/s
- T_{GO} = **24** s
- $R_{SHIP/TARGET}$ = **261.3** km
- Angle Error = Range Error / $(V_C \times T_{GO})$

Notes:
- Bold numbers are allocated values
- Shaded boxes indicate where GPS measurements are used to achieve allocations
- Brackets are target/missile track on different radar faces

KW Field of Regard Radius
11.0 mr [15.6 mr]

X 3.45 / 3

Boresight Error Requirement
9.6 mr 3σ [13.6 mr]

RSS

KW Attitude Error
4.2 mr

RSS

KW ACS Control
1.5 mr

KW Seeker to KW IMU
2.5 mr

KW IMU to 3rd IMU Transfer Alignment
3 mr

Third Stage Attitude
7.1 mr

GPS used to achieve allocation

Ship Radar to Missile Stage 3 IMU Inflight Alignment
7.1 mr **(A$_{21}$)**

Range Error = Alignment x V_C x T_{GO}

GPS used to achieve allocation

Msl Stage 3 IMU to True SBEF (ECEF)
5.0 mr   **(A$_{32}$)**

Uncorrelated Relative Target/Missile State Errors From Ship
5.0 mr [10.8 mr]

RSS

Filter Position Errors
450 m, 4.6 mr

add

Filter Velocity Errors
5 m/s x **5** s = 25 m, 0.26 mr

Time Tag Errors
(small)

20 ms

0.5 mr x 261300 m = 131 m [**3.6** mr]

Target-to-Missile Track Bias
131m, 1.3 mr [950 m, 9.7 mr]

GPS And Radar Measurement Aiding For Missile Navigation

{r_i, i = 1,2,3,4} = GPS Pseudo Range Measurements   {Δr_i, i = 1,2,3,4} = GPS Delta Pseudo Range Measurements

r_1 r_2 r_3 r_4

$\underline{P}_{M/S}$ = Position Of The Missile Relative To The Ship From AEGIS WCS Radar Track Processing

Note:
$\underline{L}_1$ and $\underline{L}_2$ are taken from the AEGIS Ship Navigation System

$\underline{P}_{M/S}$

$\underline{L}_2$ = Linear Position Change From The Initial SBEF Reference

$\underline{P}_M$

$\underline{L}_2$

OWNSHIP REFERENCE

$\underline{L}_1$ = Ship Based Earth Fixed Reference Vector

$\underline{L}_1$ SBEF REFERENCE

EARTH CENTER
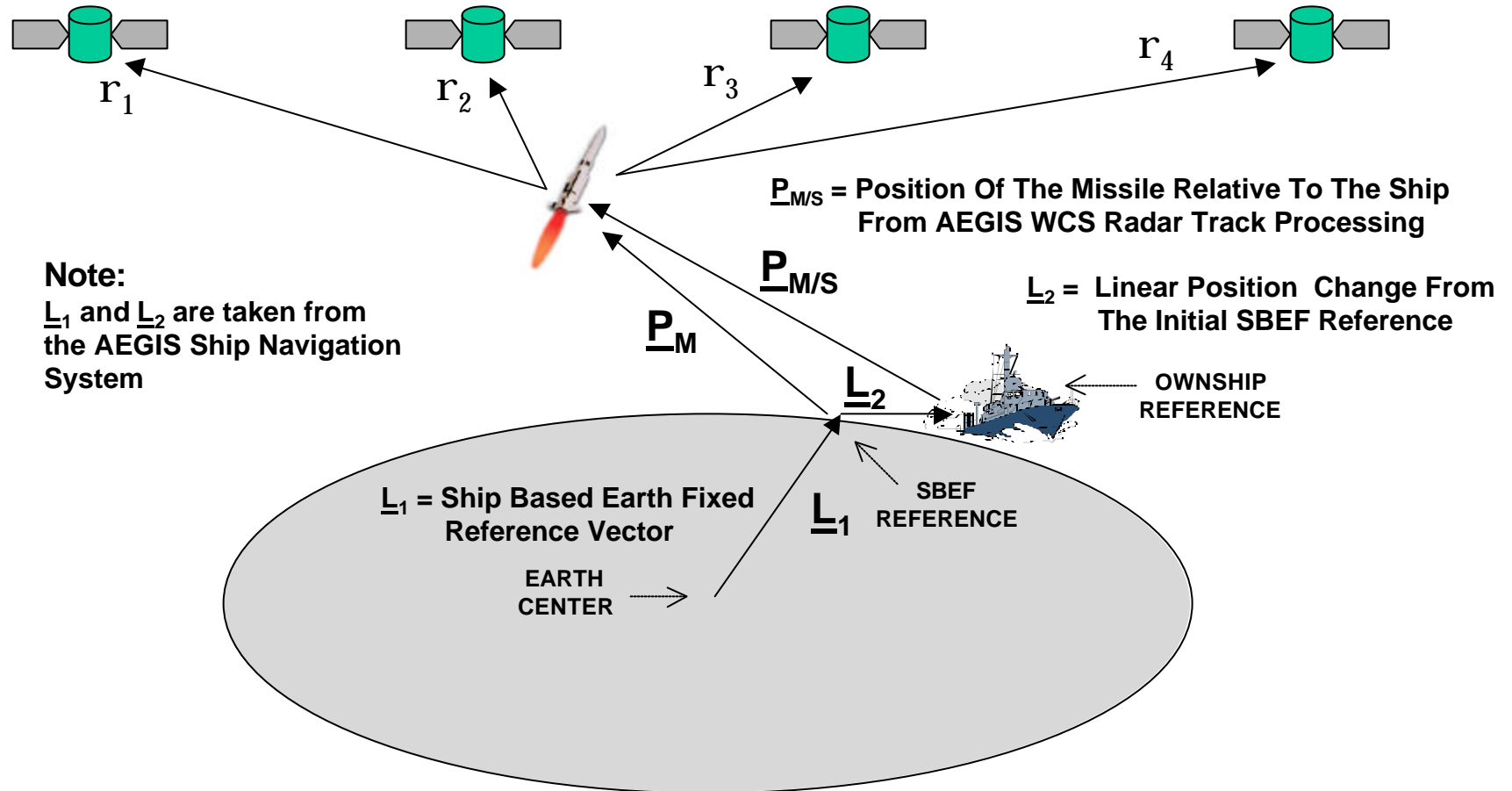
AEGIS Derived Missile Position In The ECEF Frame Requires Both Radar Measurements And Ship Navigation System Data

- 3 Position Errors

- 3 Velocity Errors

- 3 Missile Attitude Errors

- 3 Gyro Drifts

- 3 Accelerometer Biases

- 2 GPS Receiver Clock Errors (Bias & Drift)

- 3 SPY Radar Face Misalignments

- 3 Ship Initial Position Biases

Error Budget for ADOP Analyses

Navigation System Error	1s Value		
	X	Y	Z
Initial Position Error (m)	115.5	115.5	115.5
Initial Velocity Error (m/sec)	5	5	5
Initial Attitude Error (mrad)	8.72	8.72	8.72
Radar Face Misalignment (mrad)	0.8	0.8	0.8
Ship Initial Position Error (m)	1852	1852	100
Position Process Noise (m/rt-sec)	0.1	0.1	0.1
Accelerometer Random Walk (μg/rt-hz)	85	85	85
Gyro Random Walk (deg/rt-hr)	0.125	0.125	0.125
Radar Face Noise (μrad/rt-sec)	0.1	0.1	0.1
Ship Position Drift (m/rt-hr)	61.1	61.1	61.1
Radar Position Measurement Error (m)	f (range)	f (range)	f (range)
GPS Position Measurement Error (m)	10	10	10
GPS Velocity Measurement Error (m/sec)	0.3	0.3	0.3

Ship Motion Parameters	Nominal Value
Ship Speed (kts)	7
Roll Sinusoidal Amplitude (deg)	15
Pitch sinusoidal Amplitude (deg)	5
Yaw Sinusoidal Amplitude (deg)	3
Roll Sinusoidal Period (sec)	15
Pitch Sinusoidal Period (sec)	7
Yaw Sinusoidal Period (sec)	21

Note: The radar track of the missile is assumed to be constrained to SPY face 0.

ADOP Calibrated Against Detailed Navigation Simulation

- ADOP Alignment Error Comparisons with Detailed 6-DOF Navigation Simulation:

Trajectory Case	ADOP Alignment Error @ KW Ejection (mrad)			
	Radar Only		Radar & GPS	
	ADOP	NAVSIM	ADOP	NAVSIM
2	12.9	15.7	2.7	4.2
3	21.4	22.5	3.2	3.8
6	19.2	21.8	3.7	5.8
11	16.3	18.2	3.0	4.2